2009年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数学（A）

MATHEMATICS (A)

注意　☆試験時間が60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.
1 Fill in the blanks with the correct numbers.

(1) Let \( a \) and \( \beta \) be solutions of \( 3x^2 - x - 3 = 0 \).

Then \( a^2 + \beta^2 = \) \underline{\hspace{2cm}}.

(2) The solution of the inequality \(-x < x^2 < 2x + 1\) is

\[ \underline{1} < x < \underline{2} \,.

(3) Let \( \sin a = \frac{1}{\sqrt{5}} \) \((0 < a < 90^\circ)\) and \( \cos \beta = \frac{3}{\sqrt{10}} \) \((0 < \beta < 90^\circ)\).

Then \( \sin(a + \beta) = \) \underline{\hspace{2cm}}.

(4) Let \( n \) be a natural number. If \( 3^x < 2^{100} < 3^{x+1} \), then \( n = \) \underline{\hspace{2cm}}.

Use \( \log_2 3 = 0.631 \).

(5) The total number of pairs of integers \((x, y)\) which satisfy the equation

\[ x^2 - 4xy + 5y^2 + 2y - 4 = 0 \] is \underline{\hspace{2cm}}.
2 Let $f(a) = \int_0^2 |x(x-a)| \, dx$ for $0 \leq a \leq 2$.

(1) Find the function $f(a)$.

(2) Find the minimum of $f(a)$. 
Let $a$ be a real number such that $1 < a < 2$. \{a_n\} is the sequence defined by

$$a_1 = a, \quad a_{n+1} = |a_n| - 1 \quad (n = 1, 2, 3 \cdots).$$

And put $S_n = a_1 + a_2 + \cdots + a_n$.

1. Find $a_4, a_5, a_6, a_7$.
2. Find $S_2, S_4, S_6$.
3. When $n = 2m$, where $m$ is an integer $\geq 1$, express $S_n$ in terms of $a$ and $m$.
4. When $n = 2m + 1$, where $m$ is an integer $\geq 1$, express $S_n$ in terms of $a$ and $m$. 