QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2015

EXAMINATION QUESTIONS

UNDERGRADUATE STUDENTS

MATHEMATICS (A)

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
<table>
<thead>
<tr>
<th>Problem</th>
<th>Question Number</th>
<th>Your Response</th>
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1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

(1) \(2x^2 - 5x\) for the values \(1 \leq x \leq 4\) takes its maximum \([1-1]\) at \(x = [1-2]\), and its minimum \([1-3]\) at \(x = [1-4]\).

(2) Two persons \(A, B\) simultaneously toss their individual coins, and win 1 point if the head is face-up, and 0 point if the tail is face-up. The probability that the points of \(A\) exceed the points of \(B\) after three tosses is \([1-5]\).

(3) When \(a = \sqrt{5} + \sqrt{3}\) and \(b = \sqrt{5} - \sqrt{3}\), \(\frac{a}{b} + \frac{b}{a}\) is equal to an integer \([1-6]\).

(4) The negation of proposition "\(x \neq 0\) and \(y \neq 0\)" is "\(x [1-7] 0 [1-8] y [1-9] 0\)".

(5) There exist two circles that go through two points \((1, 3), (2, 4)\) and are tangent to the \(y\)-axis. Letting the radii of the circles be \(a, b\) implies that \(ab = [1-10]\).

(6) For the equation \(|2x - 1| + |x - 2| = 2\), the minimum of \(x\) is \(x = [1-11]\) and the maximum is \(x = [1-12]\).

(7) For \(\omega = \frac{1 + \sqrt{3}i}{2}\), it holds that \(\omega^5 = [1-13] + [1-14]i\), where \(i\) denotes the imaginary unit. Note that the answers are real numbers.

(8) For the sequence \(\{a_n\}\) defined by \(a_{n+1} - a_n = 2n\), \(a_1 = 0\) where \(n\) is a positive integer, the general term is \(a_n = [1-15]\).
2. A circle O is circumscribed around a triangle ABC, and its radius is r. The angles of the triangle are \( \angle CAB = a \), \( \angle ABC = b \), and \( \angle BCA = c \).

(1) The lengths of arcs AB, BC, and CA are expressed by using \( a, b, c \), and \( r \) as \([2-1]\), \([2-2]\), and \([2-3]\), respectively.

(2) The area of \( \triangle ABC \) is expressed by using \( a, b, c \), and \( r \) as

\[
\frac{r^2}{2} \left\{ \sin \left( \frac{[2-4]}{2} \right) + \sin \left( \frac{[2-5]}{2} \right) + \sin \left( \frac{[2-6]}{2} \right) \right\}
\]

(3) When \( a = 75^\circ, b = 60^\circ, c = 45^\circ \), and \( r = 1 \), the lengths of sides AB, BC, and CA are calculated as \([2-7]\), \([2-8]\), and \([2-9]\) without using trigonometric functions.
3. If a function $f(x)$ satisfies the following equation

$$\int_a^x f(t) \, dt = 3x^2 + (a + 8)x + 4,$$

then the constant $a$ is $[3-1]$ and the function $f(x)$ is $f(x) = [3-2]$. In this obtained function, the minimum of the integral $\int_a^x f(t) \, dt$ is $[3-3]$. 