2008年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2008

学科試験　問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数学(B)

MATHEMATICS (B)

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.
1 Fill in the blanks with the correct numbers.

(1) If \( \sqrt[6]{6} + \sqrt[6]{a} + \sqrt[6]{6} - \sqrt[6]{a} = \sqrt{14} \), then \( a = \underline{\phantom{00}} \).

(2) The remainder of the division of \( x^3 \) by \( x^2 - x + 1 \) is \( \underline{1} \) and that of \( x^{307} \) by \( x^2 - x + 1 \) is \( \underline{2} \).

(3) The solution of the inequality \( \log_2 (x + 1) \leq 3 \) is \( \{ \underline{1} < x \leq \underline{2} \} \).

(4) Let \( A \) be a point on the curve \( C: x^2 + y^2 - 2x - 4 = 0 \). If the tangent line to \( C \) at \( A \) passes through \( P(4,3) \), then the length of \( AP \) is \( \underline{\phantom{00}} \).

(5) Let \( x, y \) be two natural numbers such that \( x < y \), \( x + y = 96 \), and the greatest common divisor of \( x \) and \( y \) is 16. Then \( x = \underline{1} \) and \( y = \underline{2} \).
Let \( \triangle ABC \) be the triangle with \( AB = 5 \), \( BC = 4 \), and \( \angle B = 60^\circ \).

1. Find the length of \( AC \).

2. Find the radius of the circumcircle of \( \triangle ABC \).

3. Let \( D \) be a point on the minor arc of the circumcircle bounded by \( A \) and \( C \).
   Find the maximum value of the area of the quadrilateral \( ABCD \).
3 Let $F(x) = \int_1^x (3t^2 - x^2 t) \, dt$.

(1) Calculate $F'(x)$.

(2) Find the minimum of $F(x)$. 