2009年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験　問題

EXAMINATION QUESTIONS

（学部留学生）

UNDERGRADUATE STUDENTS

数学 (B)

MATHEMATICS (B)

注意　☆試験時間は60分。

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
1 Fill in the blanks with the correct numbers.

(1) Let \( \omega \) be a solution of the equation \( x^2 + x + 1 = 0 \).

Then \( \omega^{10} + \omega^5 + 3 = \square \).

(2) The constant term of \( \left(2x^4 + \frac{1}{x^3}\right)^7 \) is \( \square \).

(3) The solution of the inequality \(-x < x^2 < 2x + 1\) is \( \square < x < \square \).

(4) \( \int_0^2 x (x - 1) \, dx = \square \).

(5) If \( \frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = 6 \) and \( 0 < \theta < \frac{\pi}{2} \), then \( \tan \theta = \square \).
Denote by $D$ the domain

$$\{(x, y) \mid x \geq 0, y \geq 0\}.$$ 

Assume that a circle $C$ contained in $D$ touches the parabola $y = \frac{1}{2} x^2$ at the point $(2, 2)$ and also touches the $x$-axis. Find the radius of $C$. 
3 Let \( A, B, C \) be three points on a plane and \( O \) be the origin point on this plane.

Put \( \vec{a} = \overrightarrow{OA}, \vec{b} = \overrightarrow{OB}, \) and \( \vec{c} = \overrightarrow{OC}. \) \( P \) is a point inside the triangle \( ABC. \)

Suppose that the ratio of the areas of \( \triangle PAB, \triangle PBC \) and \( \triangle PCA \) is \( 2 : 3 : 5. \)

(1) The straight line \( BP \) intersects the side \( AC \) at the point \( Q. \)

Find \( AQ : QC. \)

(2) Express \( \vec{OP} \) in terms of \( \vec{a}, \vec{b}, \vec{c}. \)