2013 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2013

学科試験 問題
EXAMINATION QUESTIONS

(学部留学生)
UNDERGRADUATE STUDENTS

数学 (B)
MATHEMATICS (B)

注意 ☆試験時間は 60 分。
PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
1. Fill in the blanks with the correct answers.

(1) The minimum of the function \( f(x) = (2 + \sin x)(5 - \sin x) \) is ____________.

(2) If \( (2k + 1)x - (k - 2)y + 3k - 1 = 0 \) for every \( k \), then \( x = \) ____________ and \( y = \) ____________.

(3) If three straight lines \( x + 2y - 1 = 0, x - y + 2 = 0, ax - y + 3 = 0 \) meet at one point, then \( a = \) ____________.

(4) Let \( a \) and \( b \) be rational numbers. If \( \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = a + b\sqrt{6} \), then \( a = \) ____________ and \( b = \) ____________.

(5) If \( 3^x = 2^y = 5 \), then \( \frac{1}{x} + \frac{1}{y} = \log_5 \) ____________.
2. Consider the function \( F(x) = \int_{a}^{x} f(t) \, dt = x^3 - 2x^2 + x - a \quad (a \neq 0) \). Fill in the blanks with the answers to the following questions.

(1) Find \( a \).

(2) Find the range of \( x \) where \( F(x) > 0 \).

(3) Find the area of the region surrounded by the \( x \)-axis and the graph of \( f(x) \).
3. Fill in the blanks with the answers to the following questions.

(1) Find the range of \( m \) such that the equation \(|x^2 - 3x + 2| = mx\) has 4 distinct real solutions \( \alpha, \beta, \gamma, \delta \).

(2) Express the value of \( s(m) = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} + \frac{1}{\delta^2} \) in terms of \( m \).

(3) When \( m \) varies as in (1), find the range of \( s(m) \).