2016年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2016

学科試験 問題
EXAMINATION QUESTIONS

（学部留学生）
UNDERGRADUATE STUDENTS

数学 (B)
MATHEMATICS (B)

注意 ☆試験時間は 60 分。
PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
<table>
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<th>Question No.</th>
<th>Your Response</th>
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Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

1. Fill in the blanks with the correct numbers.

   (1) If \( \log_3 6 - \log_9 x = \frac{1}{2} \), then \( x = \) 

   (2) If \( \alpha, \beta \) are numbers satisfying \( 0 < \alpha < \frac{\pi}{4}, \ 0 < \beta < \frac{\pi}{4}, \ \alpha + \beta = \frac{\pi}{4} \), it follows that \( (\tan \alpha + 1)(\tan \beta + 1) = \) 

   (3) When \( x + y = \frac{2\pi}{3}, \ x \geq 0, \ y \geq 0 \), the maximum of \( \sin x + \sin y \) is \( \boxed{1} \), and the minimum of that is \( \boxed{2} \).

   (4) On the basis of the premises and the conclusions (1), (2), (3) below, fill in the lefthand blanks with 1 if the corresponding conclusion is logically derived from the premises, and with 0 if it is not.

   **Premises:** There are several three-digit numbers \( NML \) each digit of which is either 1 or 2. There are some numbers with \( N = 1 \) and other numbers with \( N = 2 \). If \( M = 2 \), then \( N = 2 \). And, if \( L = 1 \), then \( N = 2 \).
Conclusion (1): If $N = 1$, then $M = 1$.

Conclusion (2): There are no numbers with $M = 1$ and $N = 2$.

Conclusion (3): There are no numbers with $M = 1$ and $L = 1$.

(5) Consider the following diagram that consists of vertices (●) and edges (\ or - - or \ \); crosses (X) are pairs of edges whose crossing points are not vertices.

![Diagram](image)

Suppose that one can move from one vertex to another if, and only if, the two vertices are connected by a unique common edge. The number of routes that one can take from the leftmost vertex L through 6 edges and 5 intermediate vertices to the rightmost vertex R is __________.
2. Let \( r \) be a positive constant. Consider the cylinder \( x^2 + y^2 \leq r^2 \), and let \( C \) be the part of the cylinder that satisfies \( 0 \leq z \leq y \). Fill in the blanks with the answers to the following questions:

(1) Consider the cross section of \( C \) by the plane \( x = t \) \((-r \leq t \leq r)\), and express its area in terms of \( r, t \).

(2) Calculate the volume of \( C \), and express it in terms of \( r \).

(3) Let \( a \) be the length of the arc along the base circle of \( C \) from the point \( (r, 0, 0) \) to the point \( (r \cos \theta, r \sin \theta, 0) \) \((0 \leq \theta \leq \pi)\). Let \( b \) be the length of the line segment from the point \( (r \cos \theta, r \sin \theta, 0) \) to the point \( (r \cos \theta, r \sin \theta, r \sin \theta) \). Express \( a \) and \( b \) in terms of \( r, \theta \).

(4) Calculate the area of the side of \( C \) with \( x^2 + y^2 = r^2 \), and express it in terms of \( r \).

(1) 

(2) 

(3) \( a = \text{[Blank]} \) \hspace{1cm} \( b = \text{[Blank]} \)

(4) 

M−B−3
3. Let \( a \) be a number with \( a \neq 0, -1 < a < 1 \), and \( b \) an arbitrary real number. Let \( f(x) = ax + b \); moreover, let \( f^1(x) = f(x) \), and \( f^n(x) = f(f^{n-1}(x)) \) \((n = 2, 3, 4, \ldots)\). Fill in the blanks with the answers to the following questions.

(1) Express \( f^n(x) \) \((n = 1, 2, 3, \ldots)\) in terms of \( a, b, x, n \).

(2) Express \( \frac{f^n(x) - f^{n-1}(x)}{a^n} \) \((n = 2, 3, 4, \ldots)\) in terms of \( a, b, x, n \).

(3) Consider the curve \( y = \frac{f^n(x) - f^{n-1}(x)}{a^n} \) \((n = 2, 3, 4, \ldots)\) and the line \( y = ax + b \). Find the intersection point \( Q(x_n, y_n) \) of the curve and the line above, and express \( x_n, y_n \) in terms of \( a, b, n \).

(4) Calculate the limit \( \lim_{n \to \infty} f^n(x) \), and express it in terms of \( a, b, x \).

(1) 

(2) 

(3) \( x_n = \) \[
\text{(1)}
\]

\( y_n = \) \[
\text{(2)}
\]

(4) 

M–B–4