EXAMINATION QUESTIONS
UNDERGRADUATE STUDENTS

PHYSICS

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
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<th>Answer Sheet</th>
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1. Answer the following questions.

(1) A point particle with mass $m$ is thrown with speed of $v$ at an angle $\theta$ above the horizontal axis as shown in Fig. 1-1. The acceleration due to gravity is denoted as $g$. Find the distance $L$ to the point where the particle hits the ground.

\[
\begin{align*}
(a) & \quad \frac{2v^2 \sin^2 \theta}{g} \\
(b) & \quad \frac{2v^2 \sin \theta \cos \theta}{g} \\
(c) & \quad \frac{2v^2 \cos^2 \theta}{g} \\
(d) & \quad \frac{v^2 \sin^2 \theta}{g} \\
(e) & \quad \frac{v^2 \sin \theta \cos \theta}{g} \\
(f) & \quad \frac{v^2 \cos^2 \theta}{g}
\end{align*}
\]

![Diagram of particle motion](image)

Fig. 1-1
(2) There is a planet with a moon. The moon has an orbital radius of $R$, and a period of $T$. Assuming the orbit is circular, find the correct formula for the mass of the planet. Here the gravitational constant is denoted as $G$.

(a) $\frac{4\pi^2 R}{GT^2}$  
(b) $\frac{4\pi^2 R^2}{GT^2}$  
(c) $\frac{4\pi^2 R^3}{GT^2}$

(d) $\frac{R}{GT^2}$  
(e) $\frac{R^2}{GT^2}$  
(f) $\frac{R^3}{GT^2}$

(3) An ideal gas is enclosed in a container with volume $V$ and pressure $P$. The gas expands adiabatically into a vacuum. The volume changes from $V$ to $V + V'$ in this process. Find the final pressure.

(a) $\frac{V}{V + V'} P$  
(b) $\frac{V}{V'} P$  
(c) $\frac{V'}{V + V'} P$

(d) $\frac{V + V'}{V} P$  
(e) $\frac{V + V'}{V'} P$  
(f) $\frac{V'}{V} P$

(4) A point particle of mass $m$ and charge $q (> 0)$ approaches to a point particle of charge $Q (> 0)$ at a fixed position. When the distance between the two particles is $L$, the speed of the moving particle is $v$. The permittivity of the vacuum is denoted as $\varepsilon_0$. Find the minimum distance between the two particles.

(a) $\frac{QL}{q}$  
(b) $\frac{qQL}{qQ + 2\pi \varepsilon_0 mv^2}$  
(c) $\frac{qQL}{qQ + 2\pi \varepsilon_0 mv^2 L}$

(d) $\frac{qQ}{2\pi \varepsilon_0 mv^2}$  
(e) $\frac{2qQL}{2qQ + \varepsilon_0 mv^2 L}$  
(f) $\frac{2qQ}{\varepsilon_0 mv^2}$

![Diagram](image)

Fig. 1-2
(5) A light ray travels through two parallel slabs having different indices of refraction \( n_1 \) and \( n_2 \) as shown in Fig. 1-3. Which of the following choices is true regarding the relative size of indices of refraction?

(a) \( n_1 > n_2 \)   (b) \( n_1 < n_2 \)   (c) \( n_1 = n_2 \)

Fig. 1-3
2. Figure 2-1 shows a conducting bar of mass \( m \) that can slide without friction on a pair of conducting rails separated by a distance \( d \) and located on an inclined plane that makes an angle \( \theta \) with respect to the ground. The two rails are connected by a resistor. The resistance of the resistor is denoted as \( R \). A uniform magnetic field of magnitude \( B \) is directed upward, perpendicular to the ground over the entire region through which the bar moves. The bar is released from rest and slides down. The acceleration due to gravity is denoted as \( g \).

(1) Which way does the current flow, from \( a \) to \( b \) or \( b \) to \( a \)?
   (a) From \( a \) to \( b \)  (b) From \( b \) to \( a \)

Now the bar is moving along the rails at speed \( v \). The bar is always perpendicular to the rails.

(2) Which of the following is the correct formula for the induced current \( I \)?
   (a) \( \frac{v dB \sin \theta}{R} \)  (b) \( \frac{v dB \tan \theta}{R} \)  (c) \( \frac{v dB}{R \sin \theta} \)
   (d) \( \frac{v dB}{R} \)  (e) \( \frac{v dB}{R \cos \theta} \)  (f) \( \frac{v dB \cos \theta}{R} \)

(3) Besides gravity, the magnetic force acts on the bar. Find the correct formula for the component of this magnetic force along the inclined plane.
   (a) \( \frac{v dB^2 \cos^2 \theta}{R} \)  (b) \( \frac{v dB^2 \cos \theta}{R} \)  (c) \( \frac{v dB^2 \cos^2 \theta}{R} \)
   (d) \( \frac{v dB^2}{R} \)  (e) \( dB \cos \theta \)  (f) \( \frac{v dB^2}{R} \)

After a sufficiently long time the bar moves at a constant speed. At this terminal velocity \( u \) the gravity force is balanced by the magnetic force along the inclined plane.

(4) Find the correct formula for \( u \).
   (a) \( \frac{mgR}{dB \cos \theta} \)  (b) \( \frac{mgR \cos \theta}{dB^2 \cos^2 \theta} \)  (c) \( \frac{mgR \sin \theta}{dB^2} \)
   (d) \( \frac{mgR}{dB^2 \sin \theta} \)  (e) \( \frac{mgR \sin \theta}{dB^2 \cos^2 \theta} \)  (f) \( \frac{mgR \sin^2 \theta}{dB^2 \cos^2 \theta} \)
(5) Which of the following is the correct formula for the rate of work done by gravity on the bar?

(a) \( \frac{m^2g^2R \tan^2 \theta \sin \theta}{d^2B^2} \)  
(b) \( \frac{m^2g^2R \sin \theta}{d^2B^2 \cos^2 \theta} \)  
(c) \( \frac{mgR \tan^2 \theta}{d^2B^2} \)  
(d) \( \frac{mgR \tan^2 \theta}{d^2B^2} \)  
(e) \( \frac{m^2g^2R \tan^2 \theta}{d^2B^2} \)  
(f) \( \frac{mgR \sin^2 \theta}{d^2B^2} \)
3. An object of mass $M$ is hanging by a light spring of force constant $k$ from the ceiling, as shown in Fig. 3. A small ball of mass $m$ which moves vertically upward collides with the object. After the collision, the object and the small ball stick together and oscillate in simple harmonic motion. Before the collision, the object is at rest. The speed of the small ball just before the collision is denoted as $v$. The acceleration of gravity is denoted as $g$. Answer the following questions.

![Diagram of a spring-mass system with an object and a small ball colliding](image)

Fig. 3

(1) Find the amount of stretch of the spring from its natural length before the collision.

(a) $Mk$  
(b) $\frac{M}{k}$  
(c) $\frac{k}{M}$  
(d) $Mgk$  
(e) $\frac{Mg}{k}$  
(f) $\frac{k}{Mg}$

(2) Find the speed of the object and the small ball just after the small ball collides with the object and they stick together.

(a) $\frac{m}{M+m}v$  
(b) $\frac{M}{m}v$  
(c) $\frac{m}{M+m}v$  
(d) $\frac{M}{m}v$  
(e) $\frac{M}{M+m}v$  
(f) $\frac{M}{M+m}v$

(3) Find the amount of decrease of the sum of kinetic energies of the small ball and the object, before and after the small ball collides with the object.

(a) $\frac{1}{2}mv^2$  
(b) $\frac{1}{2}Mv^2$  
(c) $\frac{Mm}{2(M+m)}v^2$  
(d) $\frac{Mm}{2(M+m)}v^2$  
(e) $\frac{m(m+M)}{2M}v^2$  
(f) $\frac{m(M+m)}{2M}v^2$
(4) Find the period of the simple harmonic oscillation after the small ball and the object stick together.

(a) \(2\pi \sqrt{\frac{k}{M}}\)  
(b) \(2\pi \sqrt{\frac{M}{k}}\)  
(c) \(2\pi \sqrt{\frac{k}{M+m}}\)  
(d) \(2\pi \sqrt{\frac{M+m}{k}}\)  
(e) \(2\pi \sqrt{\frac{k}{Mg}}\)  
(f) \(2\pi \sqrt{\frac{Mg}{k}}\)

(g) \(2\pi \sqrt{\frac{k}{(M+m)g}}\)  
(h) \(2\pi \sqrt{\frac{(M+m)g}{k}}\)

(5) During the simple harmonic oscillation of the small ball and the object, which are stuck together, the spring is at its natural length when the object is at its highest position. Find the kinetic energy of the small ball just before it collides with the object.

(a) \(\frac{M(M+m)^2g^2}{km}\)  
(b) \(\frac{M^2(M+m)g^2}{km}\)  
(c) \(\frac{M(M+m)(M+2m)g^2}{km}\)

(d) \(\frac{M^2(M+m)^2g^2}{2km}\)  
(e) \(\frac{M^2(M+m)g^2}{2km}\)  
(f) \(\frac{M(M+m)(M+2m)g^2}{2km}\)
4. A cylinder is placed vertically in an atmosphere fitted with a frictionless piston of mass $M$, as shown in Fig. 4. One mole of a monatomic gas is contained in the cylinder. The cross-sectional area inside the cylinder is denoted as $A$, and the pressure of the atmosphere is denoted as $p_0$. Initially, the height of the gas in the cylinder is $h$, and the pressure of the gas is twice the pressure of the atmosphere, $2p_0$. The cylinder and the piston do not conduct heat. The gas may be regarded as an ideal gas. The acceleration of gravity is denoted as $g$, and the universal gas constant is denoted as $R$. Answer the following questions.

![Fig. 4](image)

(1) Express the mass $M$ of the piston using other quantities.

(a) $p_0Sgh$
(b) $p_0Sh$
(d) $\frac{p_0S}{g}$
(e) $\frac{p_0}{Shg}$
(f) $\frac{p_0Sh}{g}$

(2) Find the initial temperature of the gas.

(a) $\frac{p_0SR}{2}$
(b) $p_0SR$
(d) $\frac{p_0S}{2R}$
(e) $\frac{p_0S}{R}$
(g) $\frac{p_0Sh}{2R}$
(h) $\frac{p_0Sh}{R}$
(f) $\frac{2p_0SR}{R}$
(i) $\frac{2p_0S}{R}$

(3) Heat the gas slowly, and the height of the gas increased from $h$ to $\frac{3}{2}h$. Show the work done by the gas in this process.
\[(a) \quad \frac{p_0 Sh}{4} \quad (b) \quad \frac{p_0 Sh}{2} \quad (c) \quad p_0 Sh \]
\[(d) \quad 2p_0 Sh \quad (e) \quad 4p_0 h \quad (f) \quad \frac{p_0 h}{4} \]
\[(g) \quad \frac{p_0 h}{2} \quad (h) \quad p_0 h \quad (i) \quad 2p_0 h \]

(4) In the process outlined in (3) above, indicate the amount of heat given from outside:
\[(a) \quad \frac{5p_0 Sh}{5} \quad (b) \quad \frac{2p_0 Sh}{5} \quad (c) \quad p_0 Sh \]
\[(d) \quad \frac{3p_0 Sh}{2} \quad (e) \quad 5p_0 Sh \quad (f) \quad \frac{p_0 h}{5} \]
\[(g) \quad \frac{2p_0 h}{5} \quad (h) \quad p_0 h \quad (i) \quad \frac{5p_0 h}{2} \]
5. A glass tube is lying horizontally with the left end open and the right end closed using a movable piston. A speaker with a frequency of $f$ is placed on the left-hand side of the tube, as shown in Fig. 5. The position of the piston is expressed by the variable $x$ which measures the distance from the left end of the tube. Moving the piston slowly from the position at $x = 0$ to the right while the speaker is on, a large sound is heard for the first time at $x = l$. Moving the piston further, a large sound is heard for the second time at $x = 3l$. Answer the following questions.

![Fig. 5](image)

(1) A large sound is heard because a stationary wave is formed inside the glass tube. Indicate the wave length of the stationary wave.

- (a) $\frac{1}{4}l$
- (b) $\frac{1}{3}l$
- (c) $\frac{1}{2}l$
- (d) $l$
- (e) $\frac{2}{3}l$
- (f) $2l$
- (g) $\frac{3}{2}l$
- (h) $3l$
- (i) $4l$

(2) Find the expression of the speed of sound.

- (a) $\frac{1}{2}fl$
- (b) $\frac{f}{2l}$
- (c) $\frac{1}{2l}$
- (d) $fl$
- (e) $\frac{f}{l}$
- (f) $\frac{f}{l}$
- (g) $2fl$
- (h) $\frac{2f}{l}$
- (i) $\frac{2f}{l}$
- (j) $4fl$
- (k) $\frac{4f}{l}$
- (l) $\frac{4f}{l}$

(3) Indicate the distance from the left end of the glass tube to the position where the time variation of the air density is at a maximum when the large sound is heard for the second time.

- (a) $\frac{1}{2}l$
- (b) $l$
- (c) $\frac{3}{2}l$
- (d) $2l$
- (e) $\frac{5}{2}l$
(4) How does the value \( l \) change when the temperature rises?
   (a) It increases  (b) No change  (c) It decreases

(5) Increase the frequency of the speaker from \( f \) while the position of the piston is fixed at \( x = 3l \). Indicate the frequency when a large sound would next be heard.
   (a) \( \frac{1}{3}f \)  (b) \( \frac{1}{2}f \)  (c) \( \frac{1}{3}f \)
   (d) \( f \)  (e) \( \frac{2}{3}f \)  (f) \( \frac{3}{4}f \)
   (g) \( \frac{3}{2}f \)  (h) \( 2f \)  (i) \( \frac{5}{3}f \)