2009年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2009

学科試験　問題

EXAMINATION QUESTIONS

（高等専門学校留学生）

COLLEGE OF TECHNOLOGY STUDENTS

数学

MATHEMATICS

注意　☆試験時間は60分。

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.
1. Fill in the blanks with correct numbers or expressions.

1) Solve the equation $16^x - 4^x - 2 = 0$.

2) Solve the equation $\sin x + 2 \cos^2 x = 1$, $(0 \leq x < 2\pi)$.

3) Solve the inequality $x + \frac{1}{x} < \frac{1}{2}(7 - x)$.

4) Solve the inequality $\log_2(x+2) < 2$.

5) A number sequence $\{a_n\}$, ($n = 1, 2, 3, \cdots$) satisfies the following conditions. Express $a_n$ as a function of $n$.

$$3a_{n+1} = 2a_n + 1, \quad (n = 1, 2, 3, \cdots), \quad a_1 = 2.$$

6) Let $f(x) = \cos x$ and $g(x) = \sin x$. Calculate

$$\lim_{h \to 0} \frac{f(x-2h) - f(x+h)}{g(x+3h) - g(x-h)}$$
7) Differentiate the function \( e^{x \sin x} \).

8) Calculate \( \int_{1/e}^{e} \log_x x \, dx \).

9) In a single toss of two dice, find the probability that the product of the two numbers is greater than their sum.

10) Find the real value of \( a \) such that the coefficient of \( x^n \) is \( \frac{21}{2} \) in the expansion of \( \left(ax^2 - \frac{1}{ax}\right)^9 \).

11) Let A and B be the points (2,0,1) and (0,1,2), respectively. Find the point P on the line through A and B such that \( \overrightarrow{OP} \perp \overrightarrow{AB} \).

12) Let \( \alpha \) and \( \beta \) be non-real roots of the equation \( x^3 = 8 \). Find the value of \( \alpha^2 + \beta^2 \).
2) Let $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, $X = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $Y = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $Z = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

1) Find the value of $a$ which satisfies $AX = aX$.

2) Find the value of $b$ which satisfies $AY = bY$.

3) Find the values of $c$ and $d$ which satisfy $Z = cX + dY$.

4) Calculate $A^*Z$. 
Let $k$ be a positive constant, $f(x) = |x^2 - k^2|$ and $I(k) = \int_{-1}^{1} f(x) \, dx$.

1) Sketch the graph of the function $y = f(x)$.

2) Suppose $k < 1$. Express $I(k)$ as a function of $k$.

3) Suppose $k > 1$. Express $I(k)$ as a function of $k$.

4) Find the minimum value of $I(k)$ and the value of $k$ which minimizes $I(k)$.