QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2012

EXAMINATION QUESTIONS

COLLEGE OF TECHNOLOGY STUDENTS

MATHMATICS

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES.
1. Answer all questions and write your answers in the boxes provided.

1) Solve the equation $2x^3 + 3x^2 - 8x + 3 = 0$.

   \[ x = \]

2) Solve the equation $4\sin x \cos x - 1 = 0$ for $0 \leq x \leq \frac{\pi}{2}$.

   \[ x = \]

3) Solve the inequality $4^x - 2^{x+1} > 48$.

4) Solve the inequality $\log_4(2 - x) > \log_2 x$.

5) Let $\vec{a} = (2, 5)$ and $\vec{b} = (1, -1)$. Find the value of the constant $t$ which minimizes $|\vec{a} + t\vec{b}|$.

   \[ t = \]
6) Find the angle $\theta$ between the two lines $x - 2y = 3$ and $3x - y = 2$, where the range of values of $\theta$ is $0 \leq \theta \leq \frac{\pi}{2}$. 

\[ \theta = \] 

7) Calculate \[ \sum_{k=1}^{100} \frac{1}{k(k+1)}. \] 

8) Calculate \[ \lim_{x \to \infty} \frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{3x+5} - \sqrt{3x+1}}. \] 

9) Calculate \[ \lim_{x \to 0} \frac{\log(1+3x)}{x}. \] 

10) A single dice is thrown four times. Find the probability of getting a 5 twice. 

11) Find the derivative \[ \frac{dy}{dx} \] of \[ y = \log \sqrt{\frac{1+\sin x}{1-\sin x}}. \] 

12) Calculate \[ \int_{0}^{\frac{\pi}{2}} \cos^3 x dx. \]
2. Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

1) Calculate $A^2 - 5A$.

2) Calculate $A^3 - 5A^2 + A + I$.

3) Calculate $A^4 - 3A^3 - 10A^2 + A + I$. 
3 Let \( l \) be the tangent line to the curve \( y = 4 - x^2 \) at the point \((a, 4 - a^2)\), where \( a > 0 \). Denote by \( S(a) \) the area of the triangle enclosed by the \( x \)-axis, the \( y \)-axis and \( l \).

1) Find the equation of \( l \).

2) Find \( S(a) \).

3) Find the minimum value of \( S(a) \).